# Relevance of multiscale approach for the modeling of various magneto-mechanical problems

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Our time is characterized by an increasingly intensive use of actuators in extremely large fields of applications, from the largest to the smallest scales. The miniaturization of these systems adds new design constraints and requires the development of materials with controlled properties and robust models. One of the solutions being considered is the use of material exhibiting at least one strong multiphysic coupling (one of the physic being mechanics). This includes magnetostrictive materials [1], classical (SMA) or magnetic shape memory alloys (MSMA) [2], piezoelectric materials, multi-ferroic composite media, etc. One of the modeling challenges is to better describe the complex interactions observed experimentally (nonlinearity, non-monotony, irreversibly, dynamic and multiaxial effects etc ...), and to derive constitutive models with sufficient accuracy and validity range for the considered applications without requiring full field approach (micromagnetism, phase field) that still remain highly time-consuming. In this communication, the modeling of materials exhibiting magneto-elastic behavior in a reversible framework is addressed.

Showing the relevancy of scale change requires first going back to the initial foundation of Gibbs free energy density at the local scale $\alpha$ (J/m$^3$) involving mechanical and magnetic terms. The variation of Gibbs free energy $dg_\alpha$ is function of stress $\bar{\bar{\sigma}}_\alpha$ and magnetic field $\bar{H}_\alpha$ variations as control variables (1) ($\bar{\bar{\varepsilon}}_\alpha$ is the total strain and $\bar{B}_\alpha$ the magnetic induction).

$$dg_\alpha = -\bar{\bar{\varepsilon}}_\alpha : d\bar{\bar{\sigma}}_\alpha - \bar{B}_\alpha . d\bar{H}_\alpha \tag{1}$$

Small perturbation hypothesis allows the total deformation $\bar{\bar{\varepsilon}}_\alpha$ to be considered as a sum of different contributions, highlighting some specific couplings between mechanics and another physic to be defined. In the case of the target materials, classical elastic $\bar{\bar{\varepsilon}}_\alpha^e$ and free deformation $\bar{\bar{\varepsilon}}_\alpha^\mu$ associated with magnetostriction are considered leading to:

$$\bar{\bar{\varepsilon}}_\alpha = \bar{\bar{\varepsilon}}_\alpha^e + \bar{\bar{\varepsilon}}_\alpha^\mu = \boldsymbol{C}_\alpha^{-1} : \bar{\bar{\sigma}}_\alpha + \bar{\bar{\varepsilon}}_\alpha^\mu \tag{2}$$

$\boldsymbol{C}_\alpha$ is the 4$^{th}$ order local stiffness tensor. Derivation of the mechanical Gibbs free energy function involves consequently an integration of magnetostriction over the stress path making its expression complicated without any other assumption[1] (3). The magnetic part of Gibbs free energy is obtained after a Legendre transformation of the Helmholtz free energy density usually expressed as function of magnetization instead of induction (even function of magnetization) (4) [3].

$$g_\alpha^M = -\frac{1}{2}\bar{\bar{\sigma}}_\alpha : \boldsymbol{C}_\alpha^{-1} : \bar{\bar{\sigma}}_\alpha - \int_{\bar{\bar{0}}}^{\bar{\bar{\sigma}}_\alpha} \bar{\bar{\varepsilon}}_\alpha^\mu : d\bar{\bar{\sigma}}_\alpha \tag{3}$$

[1] Numerous authors simplify this expression by removing the pure mechanical part and forgetting to integrate the second part.

$$g_\alpha^\mu = \bar{M}_\alpha . \bar{\bar{P}}^0_\alpha . \bar{M}_\alpha + \bar{M}_\alpha \bar{M}_\alpha : \boldsymbol{P}^1_\alpha : \bar{M}_\alpha \bar{M}_\alpha + \bar{M}_\alpha . \bar{M}_\alpha \bar{M}_\alpha : \bar{\bar{\boldsymbol{P}}}^2_\alpha : \bar{M}_\alpha \bar{M}_\alpha . \bar{M}_\alpha - \mu_0 \bar{H}_\alpha . \bar{M}_\alpha \quad (4)$$

This expression is using a second order $\bar{\bar{P}}^0_\alpha$, 4th order $\boldsymbol{P}^1_\alpha$ and 6th order $\bar{\bar{\boldsymbol{P}}}^2_\alpha$ tensors as material dependent whose expressions are strongly correlated to material symmetries and requires assumptions for simplification. The scaling is relevant for that.

The multiscale model of a representative volume element (RVE) that is proposed involves domain and grain as subscales (figure 1). Indeed magnetization at the domain scale has a constant norm equal to the saturation magnetization $M_s$. At this scale the magnetostriction can be considered on the other hand as stress independent, allowing a simplification of the magneto-elastic coupling energy term $g_\alpha^{M\mu}$ as linearly dependent to stress and as a quadratic function of magnetization (5) [4].

$$g_\alpha^{M\mu} = -\bar{\bar{R}} : \bar{\bar{\boldsymbol{E}}}_\alpha : \bar{\bar{\sigma}}_\alpha = -\int_{\bar{\bar{0}}}^{\bar{\bar{\sigma}}_\alpha} \bar{\bar{\varepsilon}}^\mu_\alpha : d\bar{\bar{\sigma}}_\alpha = -\bar{\bar{\varepsilon}}^\mu_\alpha : \bar{\bar{\sigma}}_\alpha \quad (5)$$

$\bar{\bar{R}}$ is the second order orientation tensor and $\bar{\bar{\boldsymbol{E}}}_\alpha$ is the piezomagnetic 4th order tensor defined by 3 constants in the cubic crystallographic framework and reduced to 2 constants considering incompressibility. The constitutive behavior is assumed to follow a Boltzmann distribution allowing a statistical calculation of the domain families' volume fraction [5]. Localization and homogenization procedures, homogeneous stress and magnetic field conditions at the grain scale do complete the scheme. Some examples of applications and relevancy of stress dependent magnetostriction consideration (*morphic effect*) [3,6] will be detailed in the full paper.

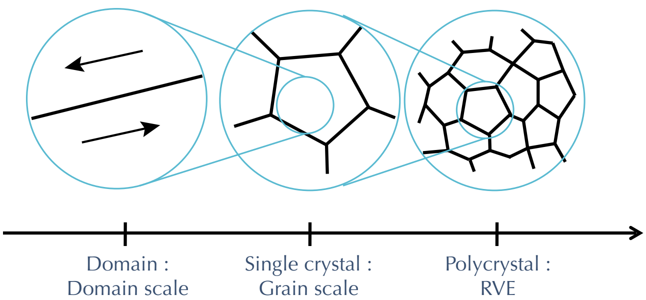


Figure 1: Detail of scales involved in the modeling approach

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